A SIMULATION ANALYSIS OF LAGGED FERTILITY ADJUSTMENTS IN DEVELOPING COUNTRIES TO EXOGENOUS MORTALITY DISTURBANCES

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Several low income countries experienced substantial mortality declines in the fifties. Thereafter, mortality gains have slowed down significantly. On the fertility side, there is increasing evidence that lagged downward fertility adjustments have probably begun.

Interacting mortality and fertility disturbances have profound implications for the growth and structure of future populations in the low-income world. As mortality gains decline and fertility declines gather momentum, population growth rates in many parts of the less-develcped world may tend to moderate levels, disproving the alarmist views of the prophets of gloom.

To gain approximate quantitative insights into the dynamics of these mortality fertility interactions, an initially stable population model based on suitable assumptions relevant for low income world is developed. This initially stable population is assumed to be subject to exogenous mortality disturbance at time t=o. The case discussed is that of a once-for-all mortality decline. Other assumptions are that birth rates respond in downward fashion to declines in death rates. The main elements of the hypothesis pertain to household family formation behaviour and are: (a) the concept of Desired Family Size; (b) household response to past mortality changes via lagged adjustment in planned fertility; (c) 'myopic' expectations about future mortality improvements. The expectations relation used is:

$$E\ y\ (t + c/t) = M\ (t + c/t).\ \ E\ y\ (t/t) \quad \ldots . (A)$$

where

$$E\ y\ (t/t) = L\ E\ y\ (t - 1/t - 1) + (1 - L)\ y\ (t - 1) \quad \ldots . (B)$$

where L = lag parameter lying between 0 and 1;
E y (t + c/t) = expected change in the force of mortality in the period (t + c), expectations formed at time t; and
y (t - 1) = actual change in the force of mortality observed in the time period (t - 1).
M (t + c/t) = Myopia factor at time t for time period (t + c) in the future.

The expectations hypothesis postulated is that currently held expectations (at time t) about mortality improvements per period in the future /myopia ignored/ are a weighted average of the lagged expectations held for the last period and the lagged observed value.

A female population whose family formation behaviour has a goal of achieving a fixed Desired Completed Family Size (DCFS) will respond to mortality changes by appropriate adjustment in their planned fertility. Mortality improvements unaccompanied by any downward adjustments in actual fertility will result in an accelerated population growth of existing numbers and further the households will discover that the number of children surviving to adulthood exceeds the quantity aimed at. Even if instantaneous and 'full' adjustments in planned fertility are made immediately following mortality disturbance and are realized, in the early stages for a time, however, the population will grow at a rate faster than previously on account of the fact that more females would survive to adulthood and higher ages than would have been the case in the absence of any downward disturbances in mortality. Thus mortality improvements will lead in the immediate future to an accelerated rate of population growth even if instantaneous and 'full' fertility adjustments accompany mortality changes. In cases in which fertility responses to mortality declines are neither instantaneous nor 'full' additional sources contributing to accelerated rate of population growth will operate. Both the magnitude and the duration of this process will depend principally upon the lag parameter. In elaborate models in which family formation takes place over a life and time span, the myopia parameter representing expected mortality improvements in the future will also be relevant in determining the sequence of the rates of population growth.

Formulae Derivation

A stable population subject to once-for-all mortality disturbance at time t.

Let a (x, t+c) denote change in the force of mortality at age x during time period (t+c). In this case, the mortality disturbance of magnitude a per period starts at time t and continues uniformly over the first time period t to t+1 and ceases at point of time t+1. Thereafter the age-specific mortality schedule at time t+1 continues unchanged. Let $u(x, t+c)$ refer to force of mortality at age x at time t+c. When the discussion is general and applies to all age groups, we will, for the sake of brevity use the notation $u(t+c)$ to refer to the force of mortality at any age at time t+c. We have:

$$u(t+g) = u(t) - ga \qquad \ldots.(1)$$

where g is a fraction lying between o and 1; and

$$u(t+c+g) = u(t) - a \qquad \ldots.(2)$$

where c is a positive integer and $o<g<1$.

Let $S(x, t-1)$ refer to before-disturbance one period survival rate schedule. When the discussion is general and applies to all age groups, we will use the notation S to refer to the pre-disturbance survival rate schedule. Let $SR(t+c)$ refer to actual one-period survival rate for any age during time period t+c, that is from time t+c to time t+c+1. Now we have:

$$SR(t) = \exp[\int_0^1 u(t+g)\, dg]$$

$$= \exp[\int_0^1 [u(t) - ag] dg]$$

$$= S. \exp[a/2] \qquad \ldots.(3)$$

$$SR(t+c) = \exp[\int_0^1 u(t+c+g) dg]$$

$$= \exp[\int_0^1 [u(t) - a] dg]$$

$$= S. \exp[a] \qquad \ldots.(4)$$

for all positive integral values of c.

Let $y(t)$ denote the change in the force of mortality at time t. We have a once-for-all decline in the force of mortality of magnitude a during period t. Hence:

$$y(t) = a$$

$$y(t+c) = 0 \quad \text{for} \quad c = 1, 2, 3, \ldots.$$

Let $Ey(t+c)$ denote the expected periodic change in the force of mortality at time t+c. Using the distributed lag relationship, we have:

$$Ey(t) = a(1-L) \qquad \ldots.(5)$$

$$Ey(t+c) = aL^c(1-L) \qquad \ldots.(6)$$

for all positive integral values of c=1, 2, ...

Expected force of mortality at various points of time (t+c) may be written as follow:

$$E\, u(t+h) = u - a - af\, L^d (1-L) \qquad \ldots.(7)$$
$$\quad d \;\; h \;\; d+1$$

where d is the integer and f the fraction part of h, for d 1 and u is force of mortality before disturbance.

The expected survival rates $ESR(t+c)$ are given by:

$$ESR(t) = \exp[-\int_0^1 E\, u(t+f)\, df]$$

$$= S. \exp[a(1-L)/2] \qquad \ldots.(8)$$

$$ESR(t+c) = \exp[-\int_0^1 E\, u(t+c+f)\, df]$$

$$= S. \exp[a + a L^c (1-L)/2] \qquad \ldots.(9)$$

If $D(t+c)$ is the planned fertility for the time period (t+c) equal to desired average number of female children born per potential mother, we have in general:

$$D(t+c). ESR(t+c) = DS = G \qquad \ldots.(10)$$

where $S = S(t-1)$ before disturbance survival rate. Hence using expression for $ESR(t+c)$, we have:

$$D(t+c) = D \exp[-a(1+L^c(1-L)/2)] \qquad \ldots.(11)$$

Let $G(t+c)$ refer to value of long-term stable population growth factor based on period (t+c) mortality and fertility schedules. We have:

$$G(t+c) = D(t+c). SR(t+c) \qquad \ldots.(12)$$

Substituting the values of $D(t+c)$ and SR we have:

$$G(t) = G \exp(aL/2) \qquad \ldots.(13)$$

since pre-disturbance long-term stable population growth rate G is equal to the product of pre-disturbance values of D and SR. In general:

$$G(t+c) = G \exp[aL^c (1-L)/2] \qquad \ldots.(14)$$

Thus two important propositions emerge from the two sets of values of $D(t+c)$ and $G(t+c)$. (1) In the periods t, t+1, ... following mortality disturbance, the survival rates are greater than the predisturbance survival rates by factor $\exp(a/2)$ for t=t and by factor $\exp(a)$ thereafter. Full fertility response would necessitate a decline in fertility by the factor $\exp(-a/2)$ during period t and $\exp(-a)$ during subsequent periods. But on account of lagged response, fertility decline factors have a sequence $\exp[-a(1-L)/2]$, $\exp[-a(1+L(1-L)/2)]$, $\exp[-a(1+L^2(1-L)/2)]$ ....$[\exp -a(1+L^c(1-L)/2)]$. Thus D, the average number of children born per potential mother falls gradually to the full adjustment level of $D \exp(-a)$, as c increases since L is less than unity. (2) The long-run stable population growth factor rises sharply at first from G in period (t-1) before disturbance to $G \exp(aL/2)$ in period t immediately following once-for-all mortality gains, and then progressively declines to G exp [a L^c (1-L)/2] during period (t+c), thus asymptotically approaching the initial level of G.

SIMULATION RESULTS

An important focus of this study is to show that notions of population 'explosion' in the low-income world are unduly alarmist. Neither the very significant declines in mortality of the 1950's can continue indefinitely, much less at those high levels; nor will the fertility response to increasing actual realized family size will be too-long delayed. A detailed analyses of census data of India (not reported here) shows that there has not so far been any significant fertility response to improving mortality experience, but there is significant evidence to show that this response is in the making and underway. One evidence of this is the rate of increase in the decennial population growth rate; this rate of increase in the growth rate was substantially lower during 1961-70 decade than it was in the preceding decade 1951-60. Simulation runs based on lagged fertility response hypothesis show that will inevitably occur as fertility declines tend to shift the family size to its desired level.

One set of illustrative simulation runs has been worked out for once-for-all mortality decline case. The initial population in both cases is assumed to be a stationary population (growth rate zero or growth factor of 1.0). The once-for-all mortality decline model has a uniform mortality fall in the first period t = 0 of magnitude a = .2202 i.e. .01 per year. Abridged Life Table l(x) column values corresponding to these assumptions are given in Annexe A Table 1. Life expectancy at birth estimates are also given in footnote (3) of the same Table. The initial t = 0 population distribution in both cases is the same and relates closely to Regional Model Life Tables population - West Females Mortality Level 7, G = 1.0 of Coale and Demeny [1] page 38.

Population projections for different values of the lag parameter L for 4 points for time t = 1, 2, 3, 4 are given in Annexe A Table 2. Important selected Characteristics of the projected populations are given in Annexe B Tables 1 through 3. Some important simulation results are discussed below.

(i) Projected populations increase over time but the rate of increase considerably slows down as time increases. This holds true for all values of L. The annual growth rate over the first period was 3.2 (per 1,000 population), increased to 5.2 in the second period, declined to 2.5 in the third period and 0.7 in the fourth period.

(ii) Projected population values were greater, greater was the value of the lag parameter.

(iii) Proportion of children in the total population which at t = 0 stood at 417 per thousand, tended to decline continuously. This was true for all L. In the FFR (Full Fertility Response Case), this proportion fell to 390 per thousand at t = 1, 352 at t = 2, 335 at t = 3 and 330 at t = 4.

(iv) The dependency ratio at first falls but then tends to rise mainly because of increasing proportions of the aged people (3+). This implies that proportion of population in potential labor force age groups rises first and then falls.

(v) Women in the child-bearing age group 1-2 increase at first but later decline; generally they stood around 30-32 percent.

(vi) Total fertility declines and asymptotically approaches the FFR value. Note over-reaction in LFR (L = .5) and NFR cases.

(viii) The long-run stable population growth factor remains at unity throughout.

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ANNEXE A TABLE 1

Abridged Life Table L(x) column Values based on Projected Mortality Disturbances

Age X	t = 0	Once-for-all Distur. t = 1,2,3,4
	l(x)	l(x)
(1)	(2)	(3)
0	1000	1000
1	583	727
2	281	437
3	84	163

Note: (1) Once-for-all Mortality Disturbance Case Based on a(0) = .2202; a(i) = 0 for i = 1, 2, 3.

(2) Life expectancy at birth underlying the above data are as follows: t = 0 (14.5 Periods); once-for-all disburbance - 18.3 periods;

ANNEXE A TABLE 2

Projected Population For Different Values of Lag Parameter L. Once-For-All Mortality Improvement Case. G=1; a=.2202 (.01 per year)

Age	t=1	t=2	t=3	t=4	Age	t=1	t=2	t=3	t=4
L=0					L=.25				
0-1	417	417	417=	417	0-1	428	420	417	417
1-2	338	377	377	377	1-2	338	387	380	378
2-3	220	274	306	306	2-3	220	274	315	308
3+	94	117	146	163	3+	94	117	146	167
Total	1068	1184	1245	1262	Total	1079	1198	1237	1270
L=.4					L=.5				
0-1	435	424	420	418	0-1	440	428	422	417
1-2	338	394	384	380	1-2	338	398	387	382
2-3	220	274	320	311	2-3	220	274	323	315
3+	94	117	146	170	3+	94	117	146	172
Total	1087	1209	1270	1279	Total	1091	1217	1279	1285
L=.75					L=1.0				
0-1	452	443	436	431	0-1	466	466	466	466
1-2	338	409	401	395	1-2	337	420	420	420
2-3	220	274	332	326	2-3	220	274	342	342
3+	94	117	146	177	3+	94	117	146	182
Total	1104	1243	1315	1328	Total	1116	1277	1374	1410

Note: (1) Initial t = 0 population distribution assumed was: Age 0-1(417), Age 1-2(302), Age 2-3(197), Age 3+(84), Total (1000)

(2) G = 1 stands for growth factor of 1 per period that is, a growth rate of zero representing a stationary population at t = 0.

ANNEXE B

Projected Population and Its Selected Characteristics for Different Values of Lag Parameter L. Once-For-All Mortality Decline Case. (G=1.0, a=.2202 (.01 per year)

Characteristics	t=0	t=1	t=2	t=3	t=4
(1)	(2)	(3)	(4)	(5)	(6)
L = 0(FFR)					
1. Population Growth Rate Per Period, Per 1000	0.00	67.7	109.2	51.3	13.7
2. Annual Average Growth Rate, Per 1000	0.00	3.2	5.2	2.5	0.7
3. Proportion Children (Age Group 0-1)	.417	.390	.352	.335	.330
4. Dependency Ratio	1.002	.916	.819	.823	.848
5. Proportion Women in Child-Bearing Age Group 1-2	.302	.316	.318	.303	.299
6. Proportion Labor Force Age Groups 1-2 & 2-3	.499	.522	.550	.548	.541
7. Total Fertility [= children per potential mother]	1.38	1.23	1.11	1.11	1.11
8. Mean Age - Periods	1.45	1.49	1.58	1.65	1.67
9. Mean Age - Years	29.0	29.8	31.6	33.0	33.4
10. Life Expectancy at Birth@ - Years	29.0	36.5	36.5	36.5	36.5
11. Long Term Stable Population Growth Factor	1.00	1.00	1.00	1.00	1.00
L = 0.5					
1. Population Growth Rate Per Period, Per 1000	0.00	91.2	115.7	50.3	5.00
2. Annual Average Growth Rate, Per 1000	0.00	4.6	5.7	2.5	.25
3. Proportion Children (Age Group 0-1)	.417	.403	.352	.330	.324
4. Dependency Ratio	1.002	.958	.811	.799	.845
5. Proportion Women in Child-Bearing Age Group 1-2	.302	.309	.327	.303	.297
6. Proportion Labor Force Age Groups 1-2 & 2-3	.499	.511	.552	.556	.542
7. Total Fertility [= children per potential mother]	1.38	1.30	1.08	1.09	1.09
8. Mean Age - Periods	1.45	1.47	1.57	1.65	1.69
9. Mean Age - Years	29.0	29.4	31.4	33.0	33.8
10. Life Expectancy at Birth@ - Years	29.0	36.5	36.5	36.5	36.5
11. Long Term Stable Population Growth Factor	1.00	1.06	.97	1.00	1.00
L = 1.0 (NFR)					
1. Population Growth Rate Per Period, Per 1000	0.00	116.0	114.4	107.6	102.6
2. Annual Average Growth Rate, Per 1000	0.00	5.5	5.4	5.1	4.9
3. Proportion Children (Age Group 0-1)	.417	.417	.365	.339	.331
4. Dependency Ratio	1.002	1.002	.840	.803	.850
5. Proportion Women in Child-Bearing Age Group 1-2	.302	.302	.329	.306	.298
6. Proportion Labor Force Age Groups 1-2 & 2-3	.499	.499	.554	.555	.540
7. Total Fertility [= children per potential mother]	1.38	1.38	1.11	1.11	1.11
8. Mean Age - Periods	1.45	1.45	1.53	1.62	1.67
9. Mean Age - Years	29.0	29.0	30.6	32.4	33.4
10. Life Expectancy at Birth@ - Years	29.0	36.5	36.5	36.5	36.5
11. Long Term Stable Population Growth Factor	1.00	1.25	1.00	1.00	1.00